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Drifting effect of electrons on fully non-linear ion acoustic waves in a magnetoplasma

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Abstract. Ion acoustic solitons affected by the drift motion of the electrons along the direction of the magnetic field have been investigated. The ranges of the existence of solitons of different velocities and the limiting value of drift velocity are reported.

1. Introduction

Extensive studies have been made of the theory of non-linear ion acoustic waves both in magnetised and unmagnetised plasmas over the last two decades. Using the reductive perturbation method in which all the non-linear terms of the equation of motion are not considered, ion acoustic solitary waves have been studied by a number of authors (Washimi and Taniuti 1966, Davidson 1972, Tappert 1972, Tagare 1972) in an unmagnetised plasma. However, the fully non-linear ion acoustic solitary waves in an unmagnetised plasma with cold ions have been investigated by Sagdeev (1966) without using the reductive perturbation technique.

Zakharov and Kuznetsov (1974) have shown the existence of three-dimensional ion acoustic solitons in a low-β magnetised plasma restricted to small amplitudes so that only some of the non-linear terms in the original equations of motion are taken into account. Also Shukla and Yu (1978) have shown the existence of finite-amplitude ion acoustic solitons propagating obliquely to the external magnetic field with a density hump for $M > k_z$. In their formulation, the ion dynamic is approximately described by the polarisation drift. Very recently Yu *et al* (1980), Ivanov (1981) and Lee and Kan (1981) have separately investigated the problem of planar ion acoustic solitary waves in a magnetised plasma with the inclusion of all the non-linear terms in the equations of motion. It is shown by Ivanov that there exist subsonic ion acoustic solitons for $k_z < M$. In this case the ion density distribution for such solitons is found to be symmetric with respect to a line passing through the peak of the soliton. On the other hand, Lee and Kan have shown the existence of ion acoustic solitons with a density hump for $k_z < M < 1$. It is also reported that no solitary waves exist with $N < 1$ and for $M > 1$.

Usually in all these investigations of ion acoustic solitons, the finite electron mass is neglected and the subsequent Boltzmann distribution for the isothermal electrons with constant temperature (T_e) has been used. In many cases, however, the properties of ion acoustic waves are considerably influenced by the drift motion of the electrons which is not negligible (for instance in DC discharges). In this case, of course, the mass of the electrons cannot be neglected and so the Boltzmann relation will not be effective. Recently, Leven and Steinmann (1979) have investigated the simple one-dimensional ion acoustic solitary waves with the effect of the drift motion of the electrons and shown the existence of solitons for $M<1$.

In this paper, we investigate analytically the planar ion acoustic solitary waves in a magnetised cold plasma affected by the drift motion of the electrons along the direction of the magnetic field. It is shown that solitons with different velocities exist in different ranges. For emphasis, the limiting value of the drift velocity is also found. The basic equations leading to the energy integral of a classical particle in a potential well has been discussed in § 2. In § 3, we have established the mathematical conditions for the existence of solitary waves. Lastly, discussions on how we drew our conclusions have been analysed in § 4.

2. Basic equations

We consider a plasma with cold ions where the electrons with constant temperature (T_e) are supposed to have initial drift motion in the direction of the magnetic field $B\hat{z}$, $\hat{z}$ being the unit vector in the z direction. All variations are set to take place in the zx plane. The equations governing the dynamics of motion under consideration are as follows:

$$\frac{\partial n_i}{\partial t}+\frac{\partial}{\partial x}(n_i v_{ix})+\frac{\partial}{\partial z}(n_i v_{iz})=0 \tag{1}$$

$$\frac{\partial v_{ix}}{\partial t}+v_{ix}\frac{\partial v_{ix}}{\partial x}+v_{iz}\frac{\partial v_{ix}}{\partial z}=-\frac{\partial\phi}{\partial x}+v_{iy} \tag{2}$$

$$\frac{\partial v_{iy}}{\partial t}+v_{ix}\frac{\partial v_{iy}}{\partial x}+v_{iz}\frac{\partial v_{iy}}{\partial z}=-v_{ix} \tag{3}$$

$$\frac{\partial v_{iz}}{\partial t}+v_{ix}\frac{\partial v_{iz}}{\partial x}+v_{iz}\frac{\partial v_{iz}}{\partial z}=-\frac{\partial\phi}{\partial z} \tag{4}$$

and

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e v_{ez})=0 \tag{5}$$

$$\frac{\partial v_{ez}}{\partial t}+v_{ez}\frac{\partial v_{ez}}{\partial z}=\frac{1}{Q}\left(\frac{\partial\phi}{\partial z}-\frac{1}{n_e}\frac{\partial n_e}{\partial z}\right). \tag{6}$$

In our formulation, the subscripts 'i' and 'e' are used to denote the corresponding quantities related to ions and electrons respectively. The highly magnetised electrons are assumed to vary only in the z direction. We have normalised densities by the equilibrium plasma density n_0, time by the inverse of the ion gyrofrequency Ω_i^{-1}, velocities by the ion acoustic speed $C_s[=(T_e/m_i)^{1/2}]$, space by the ion gyroradius $\rho_s(=C_s/\Omega_i)$ and the electric potential ϕ by T_e/e. Here $Q=m_e/m_i$ is the electron to ion mass ratio.

For a stationary solution, we consider a frame moving with the wave defined by $\xi=k_x x+k_z z-Mt$ ($M=$ Mach number $=V/C_s=$ pulse speed/ion-sound speed) where $k_x^2+k_z^2=1$.

Introducing the new coordinate ξ and using the boundary conditions $v_{ix} = v_{iz} = 0$ at $n_i = 1$ and $|\xi| \to \infty$, equations (1)-(4) can be simplified to give

$$k_x v_{ix} + k_z v_{iz} = M(1 - 1/n_i) \tag{7}$$

$$\frac{M}{n_i}\frac{\partial v_{ix}}{\partial \xi} = k_x \frac{\partial \phi}{\partial \xi} - v_{iy} \tag{8}$$

$$\frac{M}{n_i}\frac{\partial v_{iy}}{\partial \xi} = v_{ix} \tag{9}$$

$$\frac{M}{n_i}\frac{\partial v_{iz}}{\partial \xi} = k_z \frac{\partial \phi}{\partial \xi}. \tag{10}$$

Again equations (5) and (6) give, when the coordinate ξ is introduced,

$$n_e = e^{\phi}\{\exp[A(1 - 1/n_e^2)]\} \tag{11}$$

where $A = \frac{1}{2}Q(v_e' - M/k_z)^2$. In deducing equation (11), we have used the boundary conditions $\phi = 0$, $v_{ez} = v_e'$ (= drift velocity of the electrons) at $n_e = 1$ as $|\xi| \to \infty$. Equation (11) clearly gives rise to the well known Boltzmann distribution of the electrons $n_e = e^{\phi}$ when $A = 0$.

Using equations (7), (11) and the charge neutrality condition $n_i = n_e = n$ in equations (8)-(10) one can obtain

$$\frac{d}{d\xi}\left([1 - (M^2 + 2A)/n^2]\frac{1}{n}\frac{dn}{d\xi}\right) = (n-1) - \frac{k_z^2}{M^2}(n-1)(n-2A). \tag{12}$$

Multiplying both sides of equation (12) by the term in parentheses, it can be integrated once to give

$$\tfrac{1}{2}(dn/d\xi)^2 + \psi(n, M, k_z) = 0 \tag{13}$$

where

$$\psi(n, M, k_z) = \frac{n^6}{[n^2 - (M^2 + 2A)]^2}\left[(1 - k_z^2)\log n + \frac{k_z^2}{2M^2}(n^2 - 1) - \left(1 + \frac{k_z^2(1+2A)}{M^2}\right)(n-1)\right.$$
$$\left. - (M^2 + 2A)\left(1 + \frac{k_z^2(1+2A)}{M^2}\right)\left(\frac{1}{n} - 1\right) + (M^2 + 2A)\left(\frac{1}{2} + \frac{Ak_z^2}{M^2}\right)\left(\frac{1}{n^2} - 1\right)\right] \tag{14}$$

and the boundary condition $dn/d\xi = 0$ at $n = 1$ has been used. From the energy integral (13) of a classical particle in a potential well in terms of the Sagdeev potential $\psi(n)$, one can deduce the energy integral of Yu *et al* (1980) by setting the parameter A equal to zero for which $n = e^{\phi}$.

3. Criteria for the existence of solitary waves

For the existence of localised solitary wave solutions of equation (13), we need to analyse the potential $\psi(n)$ for the appropriate necessary conditions. For the solitary wave solutions, the conditions required are $\psi(1) = \psi(N) = \psi'(1) = 0$ and $\psi(n)$ must be negative between the points $n = 1$ and $n = N$, N being the maximum amplitude of the pulse.

At $n = N$, setting $\psi(N) = 0$, we can derive the non-linear dispersion relation as

$$(1-k_z^2)\log N + \frac{k_z^2}{2M^2}(N^2-1) - \left(1+\frac{k_z^2(1+2A)}{M^2}\right)(N-1)$$
$$-(M^2+2A)\left(1+\frac{k_z^2(1+2A)}{M^2}\right)\left(\frac{1}{N}-1\right)$$
$$+(M^2+2A)\left(\frac{1}{2}+\frac{Ak_z^2}{M^2}\right)\left(\frac{1}{N^2}-1\right) = 0. \tag{15}$$

The first set of conditions stated above are satisfied with the emergence of equation (15). However the essence of the conclusion sought requires the study of the behaviour of $\psi(n)$ near $n \approx 1$ and $n \approx N$. Expanding $\psi(n)$, by Taylor series near $n \approx 1$ and $n \approx N$, one obtains

$$\psi(n \approx 1) = -\tfrac{1}{2}(n-1)^2 \frac{M^2-(1-2A)k_z^2}{M^2[(1-2A)-M^2]} \tag{16}$$

$$\psi(n \approx N) = \frac{(n-N)(N-1)N^3[k_z^2(N-2A)-M^2]}{M^2(N^2-2A-M^2)}. \tag{17}$$

From (16), $\psi(n)<0$ for $n \approx 1$ if $M^2/k_z^2 > 1-2A > M^2$. Also from (17) $\psi(n)<0$ for $n \approx N$ if $N-2A > M^2/k_z^2$ when $N>1$ and if $N-2A < M^2/k_z^2$ when $N<1$.

Therefore it is seen that $\psi(n)<0$ near $n \approx 1$ and $n \approx N$ for

$$N-2A > M^2/k_z^2 > 1-2A > M^2 \tag{18a}$$

and

$$M^2/k_z^2 > 1-2A > N-2A > M^2. \tag{18b}$$

Since $A>0$, from the relations (18*a*) and (18*b*) it is therefore clear that for $N>1$, $M<k_z<1$ (or $k_z<M<1$) and $0<A<\frac{1}{2}$ and for $N<1$, $M<k_z<1$ (or $k_z<M<1$) and $0<A<\frac{1}{2}$.

Thus subsonic solitary waves exist with a density hump and density dip in our plasma model. In conformity with the conditions thus derived, the upper limit of the drift velocity of the electrons for $Q = 5\times10^{-4}$ to affect the ion acoustic solitons in our case is given by

$$v_e' - M/k_z < 44.72. \tag{19}$$

4. Discussion

With the consideration of the drift motion of the electrons in the direction of the magnetic field in a cold plasma, solitary ion acoustic waves, both with a density hump and density dip, are found to exist. These waves move with subsonic speed in different regions with different speeds which is illustrated in figure 1 for a particular value of the parameter $A = 0.4$ subject to $M<k_z<1$. When A increases to 0.5, i.e. when the drift velocity v_e' approaches its upper limit given by (19), the speed of the solitons decreases sufficiently for both cases. Even the amplitude N of the rarefactive solitary waves decreases considerably due to (18*b*) when v_e' approaches its upper limit but this is not necessarily so in case of compressive waves for (18*a*). Of course, A is found

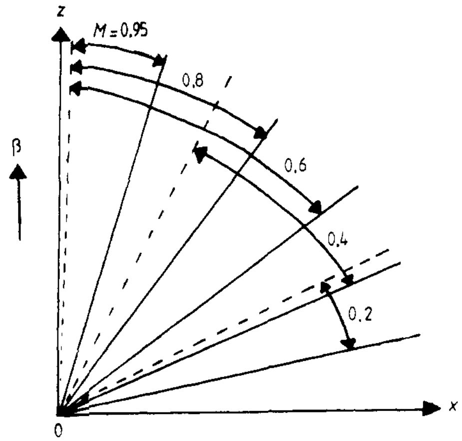

Figure 1. Regions for the existence of solitary waves with different velocities, $M(M < k_z < 1)$ for $A = 0.4$.

to change appreciably only when v_e' exceeds the value $2M/k_z$. The smaller the velocity, the greater is the deflection of the wavevector from the direction of the magnetic field for a fixed value of A. The range of the values of k_z, both for $M < k_z < 1$ and $k_z < M < 1$ with the assigned values of M and A that yield the value of $N > 1$ (or $N < 1$) from the non-linear dispersion relation (15), determined the corresponding range of the existence of compressive (or rarefactive) solitary waves. The determination of these ranges with specified velocities and the establishment of the simultaneous existence of both compressive and rarefactive solitary waves are really the new outcomes of the introduction of the drift motion of electrons. The upper limit of the values of v_e' can be determined from (19) for the least value of k_z for each range.

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